



# Numeracy is the new buzz word. What is meant by it?

**Summer School  
Utrecht University – Freudenthal Institute**

Numeracy is a human activity  
Numeracy is functional and highly practical

Kees Hoogland (HU); 24<sup>th</sup> August 2022



# Ingredients

- Awareness
- Theories
- Common European Numeracy Framework
- Rethinking basic skills

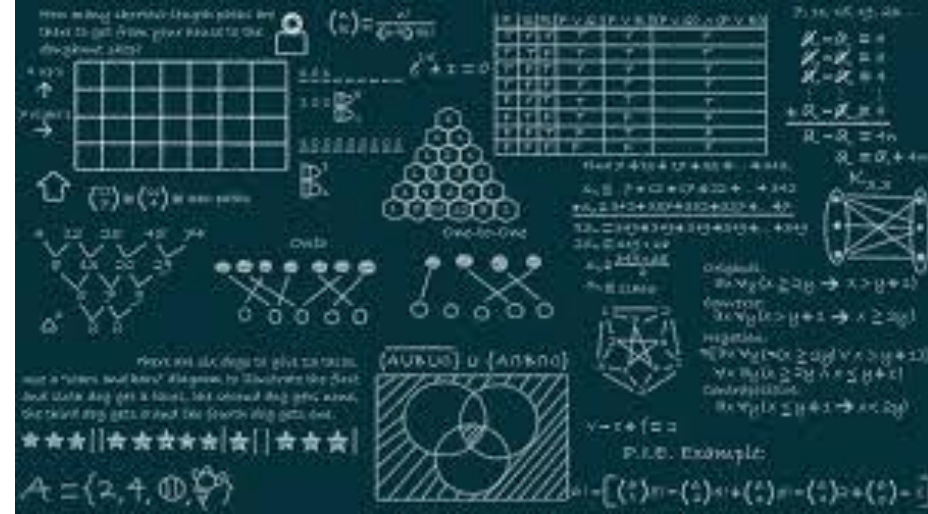


# The Mathematisation of Society

## Situations



	Renogy Wanderer 30A Li PWM Charge Controller	Renogy Rover 20A Li MPPT Charge Controller	Renogy Rover 40A Li MPPT Charge Controller
Battery Type	Sealed, gel, flooded, and lithium	Sealed, gel, flooded, and lithium	Sealed, gel, flooded, and lithium
Charge Stage	4	4	4
LCD Display	-	✓	✓
Grounding Type	Negative	Negative	Negative
Nominal system voltage	12 VDC	12V/24V DC	12V/24V DC
System Capacity	400W	200W (12 volt) / 400W (24 volt)	400W (12 volt) / 800W (24 volt)
Bluetooth Module Compatible	✓	✓	✓
Dimensions	6.5 x 4.5 x 1.8 in.	5.9 x 5.3 x 2.5 in.	5.8 x 9.4 x 2.8 in.



Microsoft Excel - Book1

File Edit View Insert Format Tools Data Window

Help Adobe PDF

A1 Expense

	A	B	C	D	E
1	Expense	Jan	Feb	Mar	
2	Phone	\$45.65	\$56.83	\$42.58	
3	Insurance	\$75.80	\$75.80	\$75.80	
4	Rent	\$750.00	\$750.00	\$750.00	
5	Totals	\$871.45	\$882.63	\$868.38	
6					

Sheet1 Sheet2 Sheet3

### Wallpaper Calculator

Wall width (m)

Wall height (m)

Wallpaper width (cm)

Roll length (m)

Pattern Repeat (cm)




# The Mathematisation of Society

Individuals acting in numeracy/mathematical situations



Mathematisation of Society - minidoc as part of Inaugural Lecture  
Kees Hoogland (2nd June 2021)

Numeracy • 10 weergaven • 1 week geleden

Mathematisation of Society - minidoc as part of Inaugural Lecture Kees Hoogland (2nd June, 2021)  
Producer: Marleen Stoker at Mokermedia marleenstoker.com

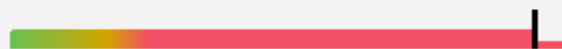
# Besmettingen



## Positieve testen

Aantal positief geteste mensen

**6.575** ↑ Waarde van 14 januari 2021



## Besmettelijke mensen

Aantal besmettelijke mensen

**140.833** ↓ Waarde van 31 december 2020

## R

## Reproductiegetal

Meest recente reproductiegetal

**0,95** ↑ Waarde van 25 december 2020



## Sterfte

Gemeld aantal personen overleden aan COVID-19 per dag

**89** Waarde van 14 januari 2021



## Verdeling positief geteste mensen in Nederland

Deze kaarten laten zien van hoeveel mensen gisteren is gemeld dat ze positief getest zijn op COVID-19, per 100.000 inwoners.

Per gemeente

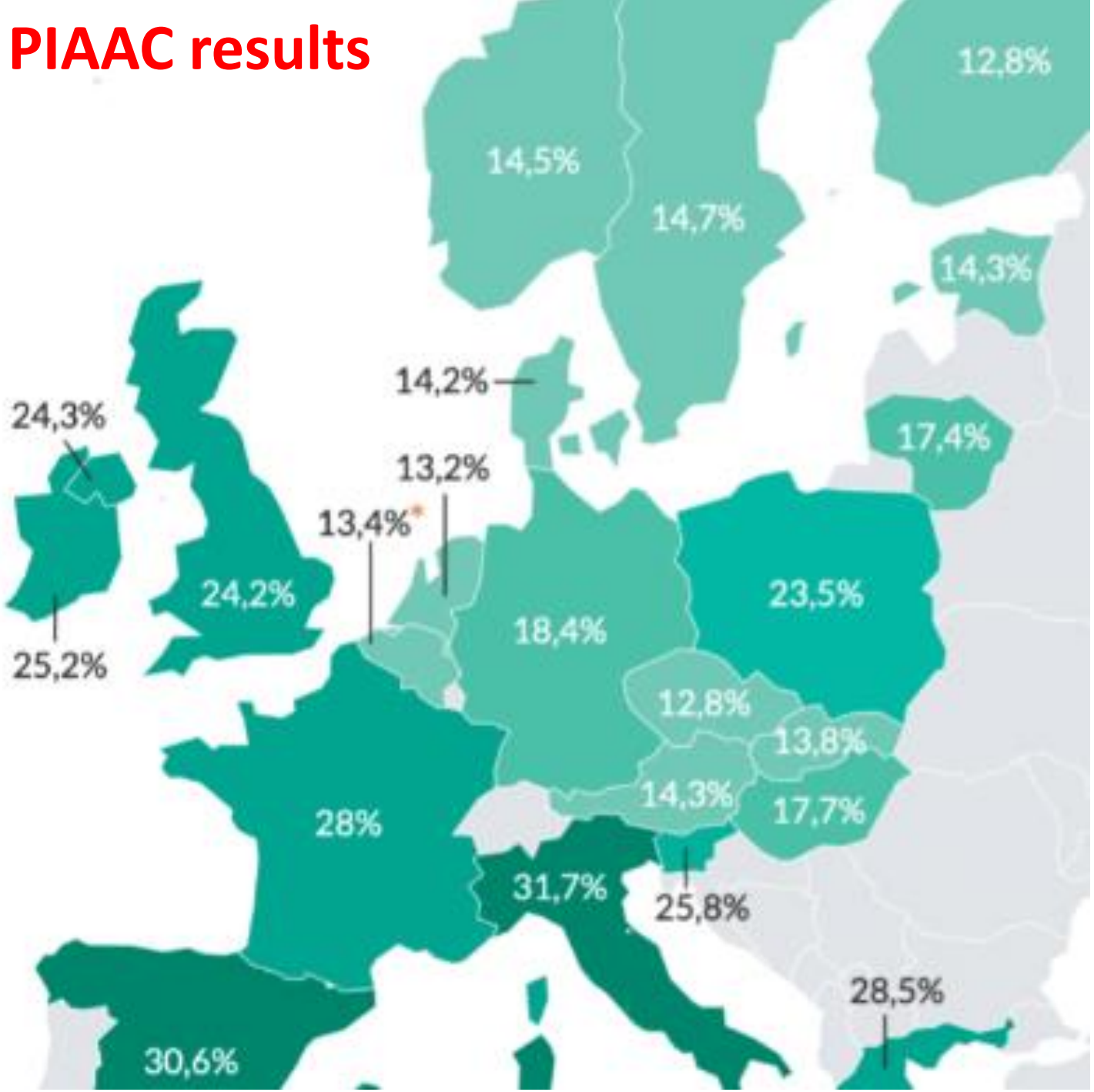
Per veiligheidsregio

Aantal per 100.000 inwoners



Waarde van donderdag 14 januari · Bron: [RIVM](#)

# PIAAC results



Source: Survey of Adult Skills (PIAAC) (2012, 2015, 2018) as indicated in Table A2.3 in Skills Matter Additional results from the survey of Adult skills (Annex A) – OECD 2019.

**Percentage of adults scoring at proficiency level 1 and below in numeracy**



OECD average 23.5%

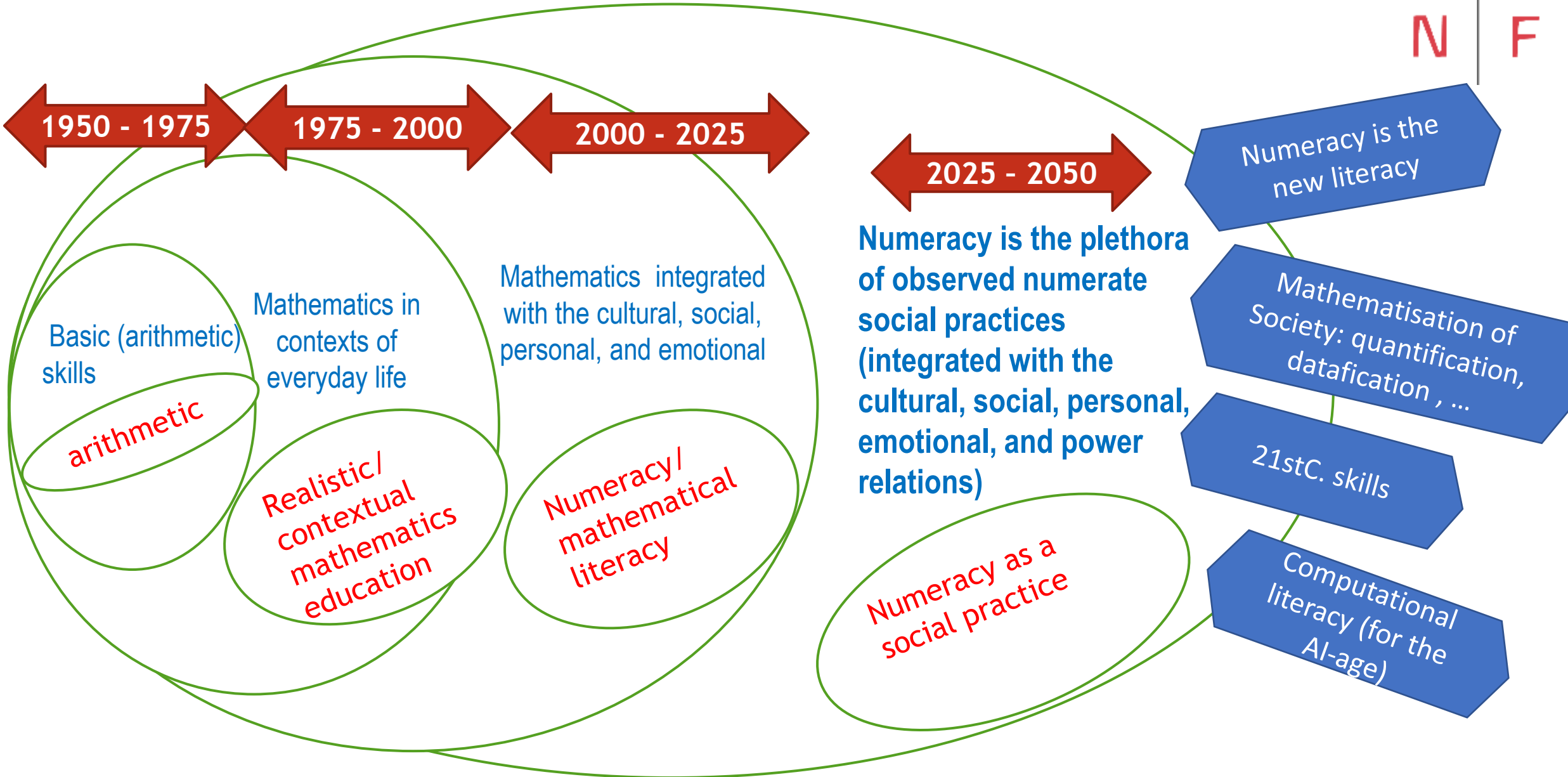


- RECORD OF BEER DISTRIBUTED FROM THE OFFICIAL STORES ON THE 12TH AND 13TH DAYS OF A MONTH, MENTIONING BEST BEER AND ORDINARY BEER, FOR THE TEMPLE, FOR THE STORE AND FOR THE HOUSE OF LU-DINGIRRA

MS 1952/39  
Beer distributed from the official stores.  
Sumer, 2080-2010 BC

# Numeracy conceptual development

C	E
N	F





# Numeracy as social practice (NSP)

“... **aggregate** of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem-solving skills that individuals need in order to **autonomously engage and effectively manage** numeracy situations that involve numbers, quantitative or quantifiable information, or visual or textual information that is based on mathematical ideas or has embedded mathematical elements”. (See Gall, 2000, p.6)

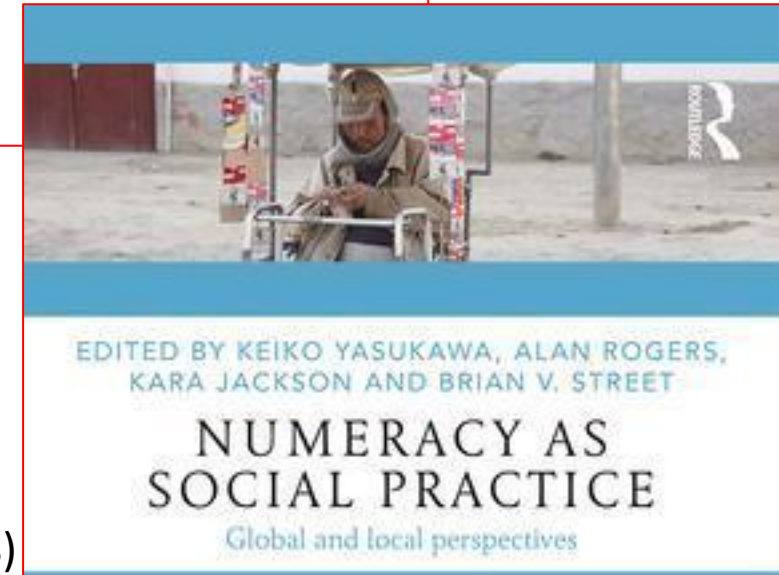
The framework acknowledges the great efforts from the past:  
ALL, IALS, PIAAC, ACER, ..., ..., ...  
It will be developed further by many.

Conceptually inspired by:

- Situated cognition
- Cultural-historical activity theory (CHAT)
- Literacy as social practice (LSP)
- Ethnomathematics

“A **social practice view of numeracy** not only takes into account the different contexts in which numeracy is practised, such as school, college, work and home, but also how people’s life and histories, goals, values and attitudes will influence the way they carry out numeracy”.

(See Oughton, 2013)



(See Yasukawa et al., 2018)

# The Numerate world 21st c. AD

## Examples in literature



- Zevenbergen (1996) Boat Building
- Evans (2000) Numeracy practices and emotions
- Coben (...) Nursing practices
- Bakker c.s (...): Airplane pilots, Bank personnel,
  - Laboratory workers
- Keogh (2018) Looking at numeracy at work
- Yasukawa e.a (Eds.) (2018): Kiwifruit orchards, Building stone walls, Managing debts
- Saló i Nevado (2021): Problem solving (cabinetmakers and farmers)

# The Numerate world 21<sup>st</sup> c.AD

## Numerate behaviour and practices

### *Cognitive processes*

### *Manifestations*

Interpretation

Product labels,  
advertisements, brochures, ....

Understanding of hidden  
algorithms

Apps, websites, ...

Valuating

Money, prices, ...

Measuring

Length, weight, ...

Estimating

Ubiquitous, ....

Critical thinking

Politics, intimidation with  
numbers

Knowing reference  
numbers

Body, country, world

...

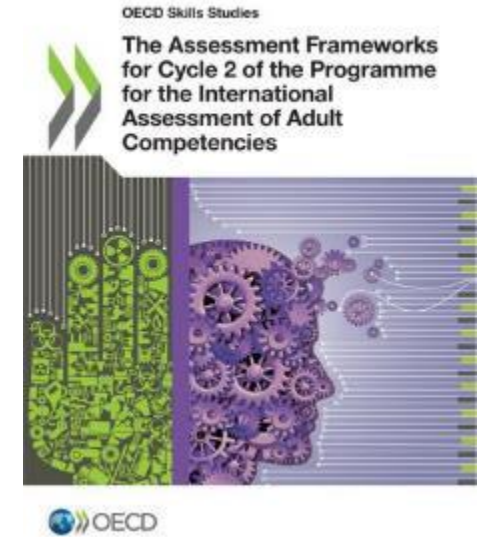
...

...

...

...

...



## Redefining basic skills



# Common European Numeracy Framework

Acknowledging that power relations play a role: exploitation, gate keeping and selection, inclusion and exclusion, gender stereotypes about handling numbers, formatting power (or terror) of school mathematics, ...

Implies:

- Explicitly take into account in developing education
- Explicitly take into account in assessing and measuring
- Explicitly discuss such topics with learners: they are after all adult citizen

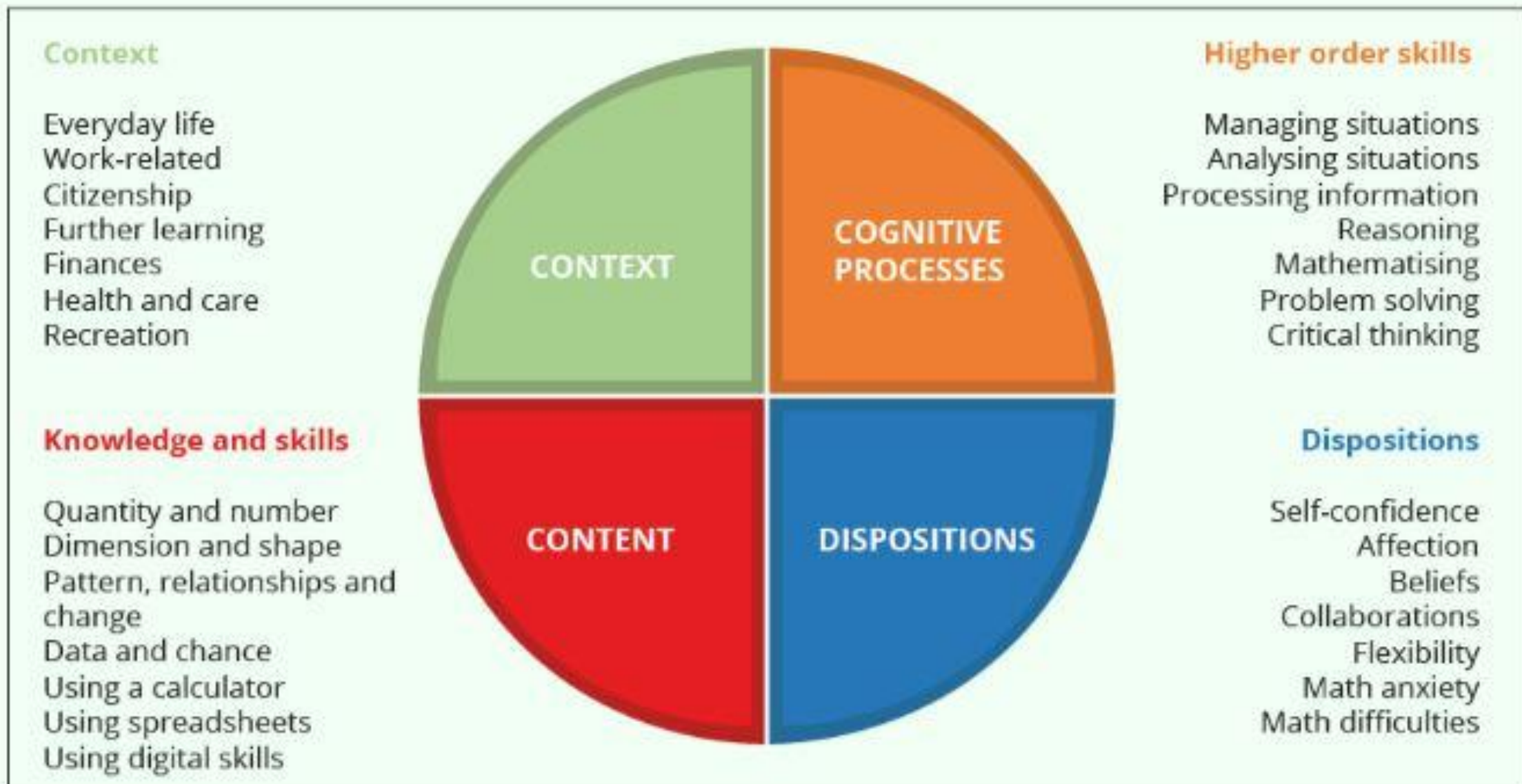
## Integral (integrative, holistic,) perspective

Acknowledging “Numeracy as a social practice”

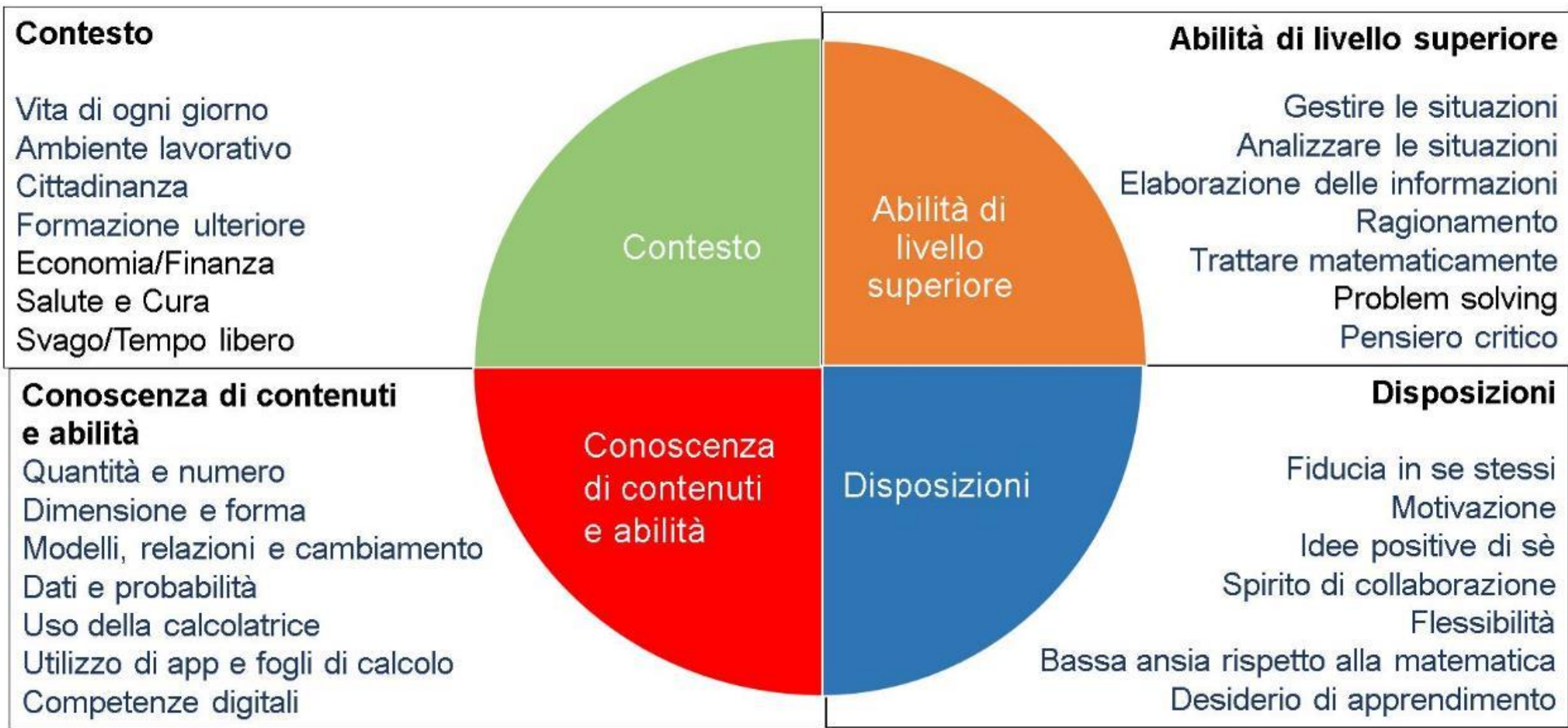
Implies: Multidimensional aspects

- cognitive and psychological aspects
- multidimensional individual profiles

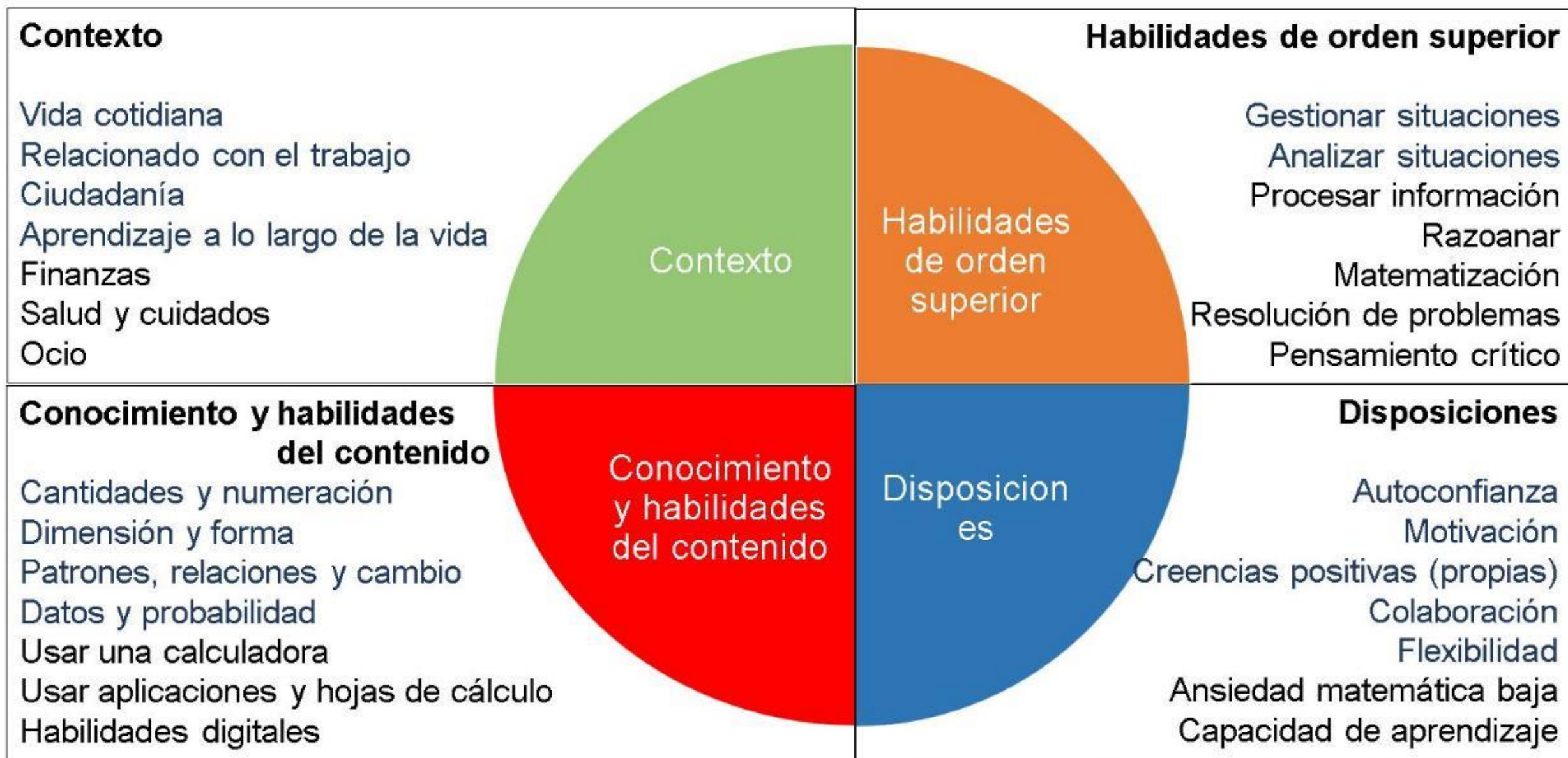
# What matters to improve numerate behavior



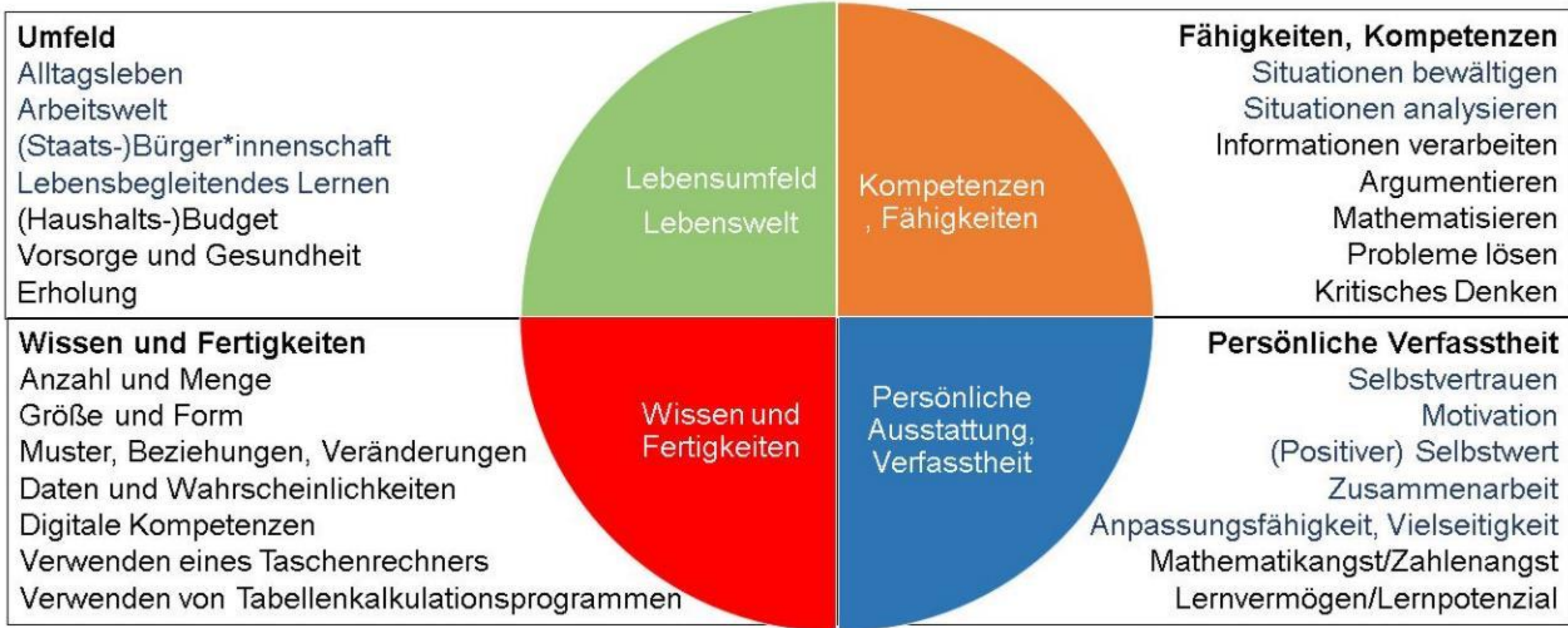
# Quali aspetti favoriscono la pratica del contare



# ¿Qué es lo que importa para mejorar el comportamiento numérico?

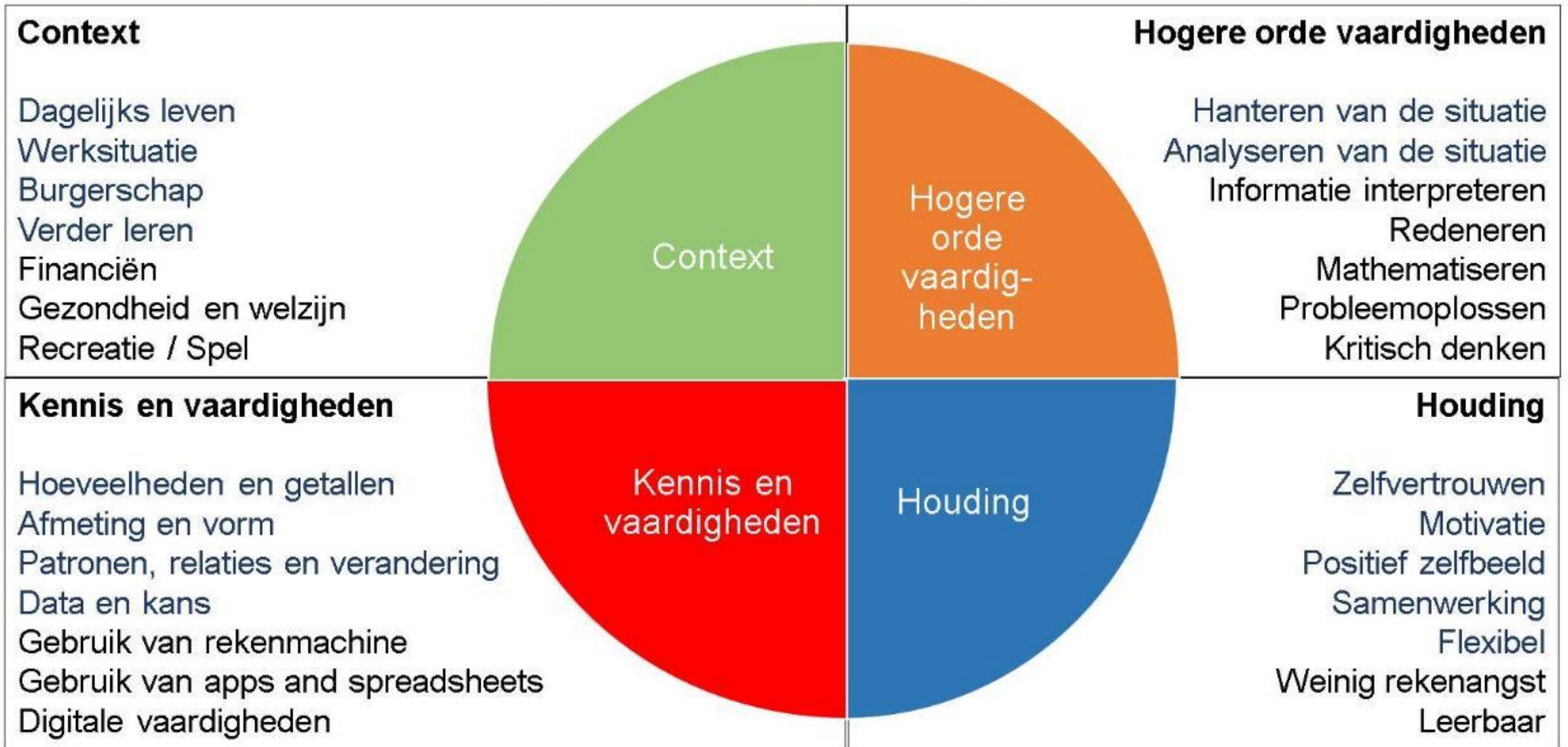


# Worauf es ankommt, den Umgang mit zahlenbezogenen Sachverhalten zu verbessern

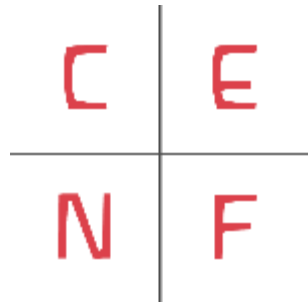




# Factoren die gecijferd gedrag verbeteren



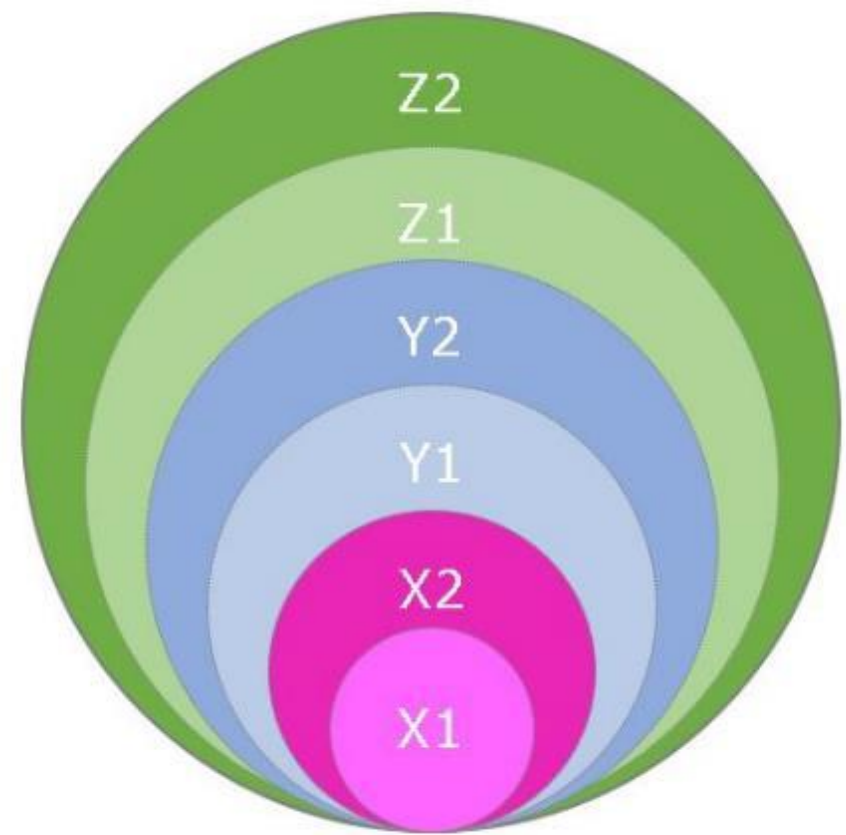
# CENF - Overall levels (= categories $\neq$ thresholds)



Z  
Specialized  
societal and  
work situations

Y  
Societal and  
regular work  
situations

X  
Daily-life  
situations



©CENF, 2021

- Z2** ... Manage situations which require interpreting multiple types of mathematical information where considerable translation or interpretation is required to come to a solution, make inferences, and develop or work with mathematical arguments or models.
  - ... Understand and use complex representations and abstract and formal mathematical and statistical ideas, possibly embedded in applications, tools and texts.
  - ... Justify, evaluate, and critically reflect upon problem assumptions, solutions, and choices.
  - ... Use sophisticated statistical and mathematical software in complex professional situations.
- Z1** ... Manage situations which require analysis and more complex reasoning about quantities and their selfish and change, spatial relationships and change, proportions, and formulas.
  - ... Understand and use a broad range of mathematical information that maybe complex, abstract, or embedded in unfamiliar contexts. These tasks involve undertaking multiple steps and choosing relevant problem-solving strategies and processes.
  - ... Communicate arguments and well-reasoned explanations for answers or choices.
  - ... Use standard statistical and mathematical applications for all kind of work situations.
- Y2** ... Manage situations which require several steps to interpret the situation and involves the choice of problem-solving strategies and relevant processes, such as the application of number, sense and spatial sense, recognizing and working with mathematical relationships, patterns, and proportions expressed in verbal or numerical form.
  - ... Identify and act on mathematical information that maybe less explicit, embedded in familiar and unfamiliar contexts, tools and applications and use them to decide and actively communicate.
  - ... Use various applications for work, householding, and leisure.
- Y1** ... Manage situations which require the application of two or more steps or processes involving calculation with whole numbers and common decimals, percentages, and fractions; simple measurement and spatial representation; estimation.
  - ... Identify and act on mathematical information and ideas embedded in a range of familiar contexts, tools, and applications consisting of relatively simple data and statistics in texts, tables and graphs and use them to decide and further communicate.
  - ... Use some standard applications for work, householding, and leisure.
- X2** ... Manage everyday life situations which require one-step or simple processes involving counting, sorting, estimation, addition or subtraction, operations needed to decide and further communicate.
  - ... Interpret elements of simple or common numerical, graphical or spatial representations and use them to decide and further communicate.
  - ... Use familiar and familiar digital devices, use simple processes and some default workbooks.
- X1** ... Manage concrete, familiar situations where the mathematical problem is explicit and familiar and processes required are simple.
  - ... Identify and act on simple numerical representations and use them to decide.
  - ... Perform processes involving either counting, sorting, and basic arithmetic operations with whole numbers or money.
  - ... Use some digital devices occasionally.

[www.cenf.eu](http://www.cenf.eu)

# Educational activities

- Numeracy conversation
- Counteract math anxiety (Talk about it!)
- Exploring the quantitative world around you
  - e.g., Using pictures of real life situations
- Working on awareness of personal successful numerical behaviour
- Discuss and work on themes (finance, health, climate, ....)
- Discuss and work on numbers in media, news, advertisements

X1 /  
X2

Y1 /  
Y2

# Educational activities

- Thematic courses/meetings
  - Better trading on E-Bay
  - Educational games
  - Budgeting, planning, saving
  - Cooking
  - Et cetera
- Discussing weekly experiences: critical dialogue
- Adult education: integrate into language lessons and into language support...
- Adult education: integrate into digital skills courses





# Redefining Basic Skills

**Absolutely fine those broad holistic ideas and connecting with reality, this is very appropriate for our learners who will use it in a functional and practical way but first, of course, we need to**

**"explain"**

**"re-teach"**

**"repair"**

**"practice"**

**"remedy"**

**these basic calculational skills**

**"which they don't have"**

**"which they have not learned"**

**"which they didn't maintain enough"**

**"which they don't learn/teach anymore in primary school"**

**Persistent  
"calculational"  
paradigm**



# Catastrophic teaching of basic skills

## 1. Learn - practice - never use

Demotivation, alienation, loss of meaning.

These skills disappear or become a superficial memory item ("They never taught me this." "I can't remember this, or maybe vaguely" (but it arouses anxiety nevertheless)

## 2. Learning – practicing – only use in test or exam

Teaching to the test, learning to the test, fixed mindsets, no ownership, no personal development. ("Tell me exactly what to do.").

These skills do not last or badly. After test or exam rapid decrease in skills. Math anxiety is increased.

# Teaching skills to use

- **Learning – practicing – using functionally**
  - In daily life: indoors and outdoors
  - In vocational situations
    - General vocations: tables, dimensions, spreadsheets
    - Specific vocations: formulas, apps,
  - In games and digital games
  - In (social) media
  - In concrete situations
  - With concrete materials

**Ultimately aiming at “unconscious/unnoticed” use.**

# 1900 - 1975

- Rise of mass education
- Industrialisation
- Capitalism: economic transactions
- Standardized procedures to calculate
- Calculations with pen-and-paper
- Decimal metric system



At school: teaching of fixed procedures - focused on mathematical structure and not on functional use

# 1900-1975

- Focus on **basic facts** in **formal** notations
  - $7 \times 9 =$
  - $12 + 9 =$
  - $34 - 18 =$
  - $35 : 7 =$

$1 \times 2 =$   
 $2 \times 2 = 4$   
 $3 \times 2 = 6$   
 $4 \times 2 = 8$   
 $5 \times 2 = 10$   
 $6 \times 2 = 12$   
 $7 \times 2 = 14$   
 $8 \times 2 = 16$   
 $9 \times 2 = 18$   
 $10 \times 2 = 20$

$1 \times 3$   
 $2 \times 3$   
 $3 \times 3$   
 $4 \times 3$   
 $5 \times 3$   
 $6 \times 3$   
 $7 \times 3$   
 $8 \times 3$   
 $9 \times 3$   
 $10 \times 3$



$1 \times 6 =$   
 $2 \times 6 =$   
 $3 \times 6 =$   
 $4 \times 6 =$   
 $5 \times 6 =$   
 $6 \times 6 = 36$   
 $7 \times 6 = 42$   
 $8 \times 6 = 48$   
 $9 \times 6 = 54$   
 $10 \times 6 = 60$

$1 \times 7 =$   
 $2 \times 7 =$   
 $3 \times 7 =$   
 $4 \times 7 =$   
 $5 \times 7 =$   
 $6 \times 7 =$   
 $7 \times 7 = 49$   
 $8 \times 7 = 56$   
 $9 \times 7 = 63$   
 $10 \times 7 = 70$

$1 \times 8 =$   
 $2 \times 8 =$   
 $3 \times 8 =$   
 $4 \times 8 =$   
 $5 \times 8 =$   
 $6 \times 8 =$   
 $7 \times 8 =$   
 $8 \times 8 = 64$   
 $9 \times 8 = 72$   
 $10 \times 8 = 80$

$1 \times 9 =$   
 $2 \times 9 =$   
 $3 \times 9 =$   
 $4 \times 9 =$   
 $5 \times 9 =$   
 $6 \times 9 =$   
 $7 \times 9 =$   
 $8 \times 9 =$   
 $9 \times 9 = 81$   
 $10 \times 9 = 90$

**Basic calculation facts are executed by heart/instantaneously only when they are automated.**

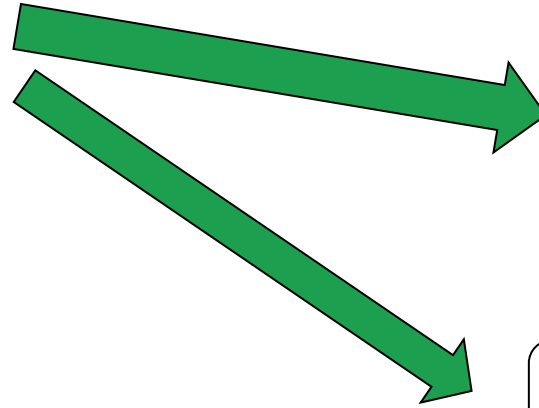
**Automated = learned->practiced->used (a lot)**

This is not the same as Memorizing



# 1900- 1975

- Were these (formal) basic facts like  $6 \times 8$  and  $13 - 9$  widely used?



**Board Games**

**Estimations**

Especially in all those pen-and-paper calculations that were needed to perform larger calculations with pen-and-paper.

$$\begin{array}{r} 789 \\ 56 \\ \hline \end{array} \times$$

$$\begin{array}{r} 512.693 \\ 45.678 \\ \hline \end{array} +$$

$$\begin{array}{r} 123 \\ 56 \\ \hline \end{array} -$$

$$\begin{array}{r} 35.750 : 12 = \\ \text{of} \\ 12 / 35.750 \backslash \end{array}$$

**That was at that time a very functional use in study, profession and daily life**

# G. S. Ruiter's Schaatsen- en Gereedschappenfabriek

Kantoor: Jacob Marlotstraat 24  
 Werkplaats: Van Schone 53  
 Bankrekening: Kingma's Bank N.V. Leeuwarden  
 Giro No. der bank: 4240  
 Telefoon no. ....

SUIJP- EN REPARATIE-INRICHTING - SMEDERIJ

HUIZUM, 5 April 1950

FACTUUR voor de firma M. W. G. ...

N<sup>o</sup> 1276

Reclames binnen 8 dag na ontvangst goedkeuren

Korrelend. d. d. 1/4 1950 aan de heer ...		
Order n <sup>o</sup> 4557		
6	Korrelend. d. d. 1950 1/2 6 dm	3.90
6	Felom 1/2 dm	4.20
6	Felom 22x13 1/2 1 dm	4.00
16	Breukelend. d. d. 20x13 1/2 12 dm	16.20
12	Felom 20x16 1/2 12 dm	18.00
12	Felom 17x16 1/2 12 dm	17.20
12	Felom 17x16 1/2 12 dm	20.40
5	Rand. d. d. 19x13 1/2 6 dm	19.50
3	Felom 1/2 dm	1.00
2	Breukelend. d. d. 26 1/2 12 dm	3.60
Alomente		108.50
1 stuk korrelend. d. d. 1/2 12 dm		96.20

*[Handwritten signature]*

7 APR 1950

Betaling binnen 8 dagen door overschrijving op onze bankrekening bij KINGMA'S BANK N.V. LEEUWARDEN, Giro No. 4349

# HOTEL Braams

GIETEN  
 Telefoon (05926) 2 41 - 2 42  
 Postgiro 83 85 96

Eigenaar J. T. H. Rijnberg

Tafel Nr.: 4 Kellner: *[Handwritten name]*

Aperiti f	5.45
2 Oostvaardseep	4.00
1 Solle Fide	10.50
1 Solle Donsil	8.50
2 Wijn	2.60
<hr/>	
106 Dinsu	31.05
	46.50
	35.70
<hr/>	
2 koffie (incl.)	1.50
	37.20

IN NATIONALE ALPHABETEN 6949

Datum:

000641-35

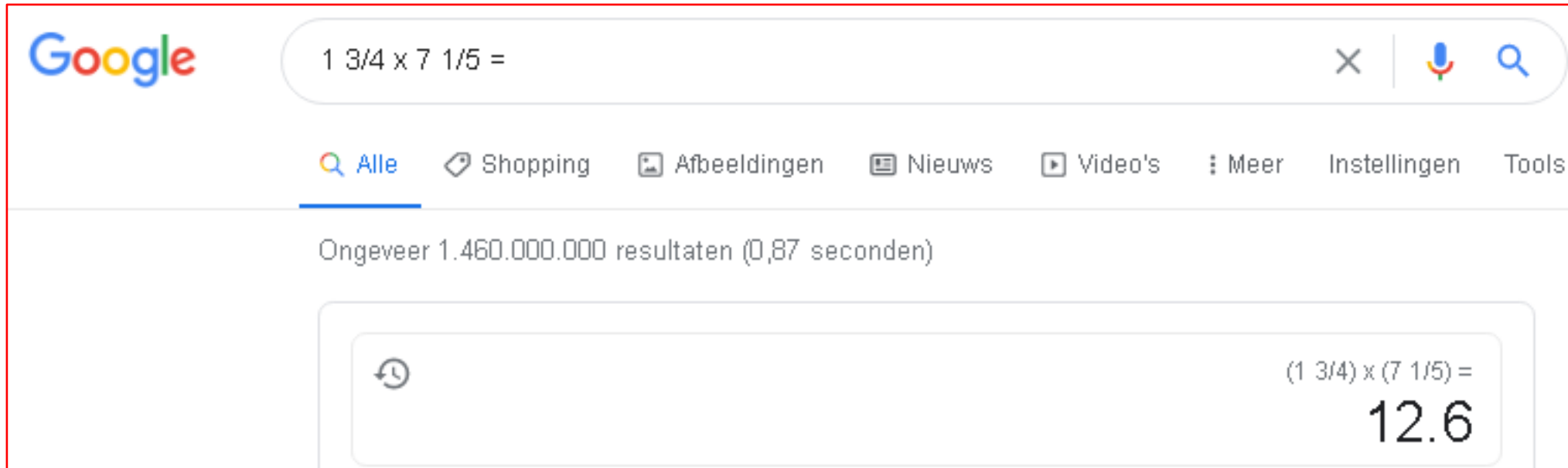
196

# 1975 - 2050

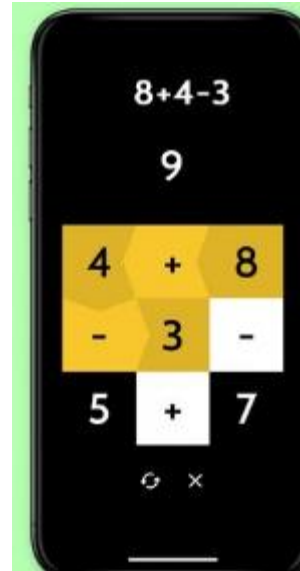
- Calculators
- Computers
- Models (AEX, weather, ...)
- Digitization of services
- There's an App for that....



# Basic skills in 2050



A screenshot of a Google search interface. The search bar contains the mathematical expression  $1 \frac{3}{4} \times 7 \frac{1}{5} =$ . Below the search bar, navigation options include "Alle", "Shopping", "Afbeeldingen", "Nieuws", "Video's", "Meer", "Instellingen", and "Tools". The search results show "Ongeveer 1.460.000.000 resultaten (0,87 seconden)". A large white box displays the calculation  $(1 \frac{3}{4}) \times (7 \frac{1}{5}) = 12.6$ .





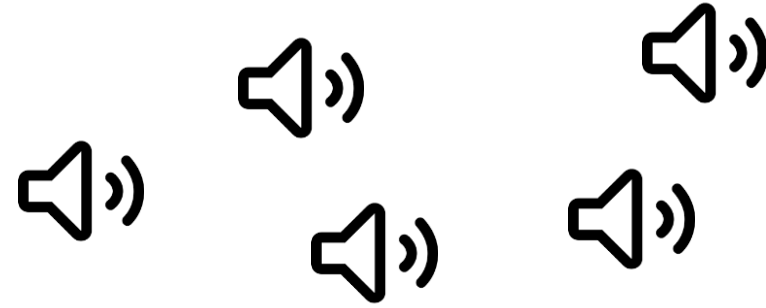
# 1975 – 2050 Basic facts

## Unnoticed:

Games en digital games



## Auditory:



## Visual:



## Estimate $\approx$

$$7 \times 11,9 \approx$$

$$12 \times 500.000 \approx$$

$$6 \times 125 \approx$$

$$1000 : 71 \approx$$

$$500 \times 7 \text{ MB} \approx$$

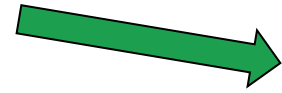
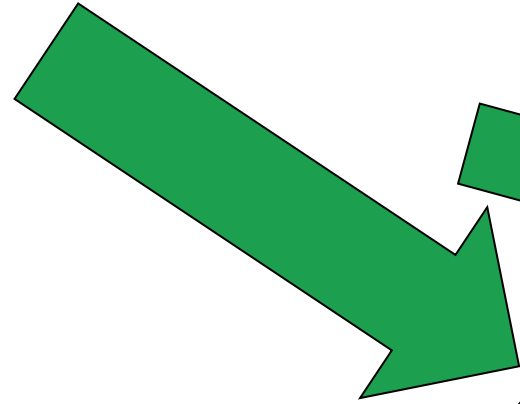
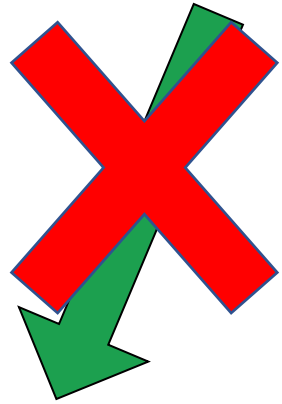
1975 - 2050

Are the basic facts like  
6 x 8 and 13 - 9 still widely  
used?

# 1975 - 2050



Are these basic facts (6 x 8 and 13 - 9) widely used?



**(digital) games**

**Proportional reasoning**

**Large calculations are done with "tools"**

234	512.693	123	35.750 : 12 =
<b>16</b>	45.678	56	of
----- x	----- +	-----	12 / 35.750 \

**Estimating, global arithmetic, order of magnitude, ...**

≈

6 boxes of € 11,95 ≈  
room of 6.3 by 4.8 ≈  
1.200 x 50.000 ≈

- Extra basic facts**
- 7 x 50
  - 6 x 25
  - 200 : 5
  - 1000 x 1000

**Functional use**

Use in recognizing and using proportions,  
and in calculations with percentages

persons	1	4	...	120
quantity in grams	..	600	1000	...

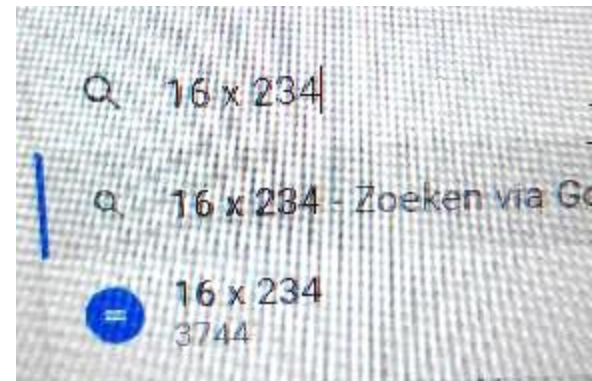
If you know a set of two, then  
you know the all!

Number...	...	75	...	250	...
%	1	...	50	100	121

Number...	...	...	79,95	...	129,95
%	1	21		100	121

# 1975 – 2050 Tools

- Use of tools is permitted !!
- Dealing well and wisely and critically with a calculator / calculation app is a skill.  
So, you have to practice it a lot and consciously!!
- And then use it a lot and use it critically.
- Where are the learning materials who support the learning of how to use a calculator.
- Use calculator properly (PC, phone, web-based)?
- Use of Google etc. in a proper and sensible way?



## Summary redefining basic skills

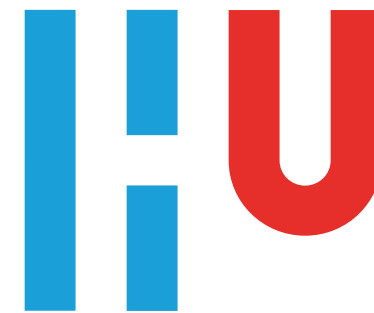
- Learning and practicing math facts, preferably informal, in (digital) games, visual, auditory, ...
- Using calculation facts for estimates, ratio tables, conversion, ...
- Learn to master tools for calculations and use them critically (and that does not happen automatically).

# Key message of this presentation

- Numeracy, Literacy and Digital Competences are crucial and connected basic skills for individuals coping with the digitalised and technologised 21<sup>st</sup>-century society.
- This will work out if and only if Numeracy, Literacy, and Digital Competences are defined and implemented as multifaceted, social and 'holistic' concepts which are intertwined, and integrated in human behaviour.
- This means that **numeracy** in the major policies of almost all European countries needs a serious upgrade regarding awareness, content, professional development, and provisions. A Common European Numeracy Framework can be instrumental to this.
- Systematically acknowledge multidimensionality when dealing with numeracy (research, teaching, professional development, ....)
- Redefine basic skills in (more) relevant cognitive processes and their manifestations.

For information, collaboration, and  
comments, please contact Kees Hoogland

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<https://www.gecijferdheid.nl/kees-hoogland-appointed-professor-of-mathematical-and-analytical-competences-of-professionals/?lang=en>

- Programme manager of Erasmus+ project: Common European Numeracy Framework
- Member of the OECD - Numeracy Expert Group - PIAAC 2nd cycle
- Trustee of Adults Learning Mathematics – A research Forum
- Fellow of the International Society for Design and Development in Education
- Chair of the Thematic Working Group - Adult Mathematics Education - at CERME 12 (Bolzano, Italy, 2-6 February 2022)

Just published:

- ALM: key-note <https://www.gecijferdheid.nl/adult-numeracy-practices-imperative-implications-for-education/>
- Springer: National Reflections on the Netherlands Didactics of Mathematics: <https://link.springer.com/book/10.1007/978-3-030-33824-4>
- ZDM: "Computer-based assessment of mathematics into the twenty-first century: pressures and tensions" <https://rdcu.be/Oz4e>



## Mathematisation of Society - minidoc as part of Inaugural Lecture Kees Hoogland (2nd June 2021)

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Mathematisation of Society - minidoc as part of Inaugural Lecture Kees Hoogland (2nd June, 2021)

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